The Gordon Model: Derivation & Relation To P/E Multiple

The Gordon Model assumes constant growth to perpetuity from a starting earnings or cash flow (in this paper I use the terms interchangeably). We will term the first cash flow as period s (start) and the prior year's earnings as E. With a growth rate of g, forecast earnings in the first year is E(1+g). The Net Present Value (NPV) of the series of cash flows would appear as follows, assuming End-of-Year cash flows for simplicity:

[1] $$NPV = \frac{E(1+g)}{(1+r)^s} + \frac{E(1+g)^2}{(1+r)^{s+1}} + \ldots + \frac{E(1+g)^\infty}{(1+r)^\infty}$$

Factoring out earnings,

[2] $$NPV = E\frac{(1+g)}{(1+r)^s} + \frac{(1+g)^2}{(1+r)^{s+1}} + \ldots + \frac{(1+g)^\infty}{(1+r)^\infty}$$

Multiplying both sides of the equation by $\frac{(1+g)}{(1+r)}$,

[3] $$\frac{1+g}{1+r}NPV = E\frac{(1+g)^2}{(1+r)^{s+1}} + \ldots + \frac{(1+g)^\infty}{(1+r)^\infty}$$

We must assume in this model that the perpetual growth rate, g is less than the discount rate, r. With that assumption, the term on the extreme right goes to zero as the exponents go to infinity. Therefore the only term that remains on the right hand side of the equation upon subtracting [3] from [2] is the first term in the square brackets, and we come to:

[4] $$NPV\left[1 - \frac{1+g}{1+r}\right] = E\frac{(1+g)}{(1+r)^s}$$

The term in square brackets simplifies to $\frac{r-g}{1+r}$. Multiplying both sides of the equation by $\frac{1+r}{r-g}$, the equation becomes:

[5] $$NPV = \frac{E(1+g)}{(1+r)^{s-1}(r-g)}$$

Finally, we can move the $(1+r)^{S-1}$ to a separate denominator, and we arrive at our final solution for a Gordon Model with end of year cash flows:

Gordon Model: End of Year Cash Flows

[6] $$NPV = \frac{\frac{E(1+g)}{(r-g)}}{(1+r)^{s-1}}$$

Another way of expressing [6] is E(1+g) $\times \frac{1}{r-g} \times \frac{1}{(1+r)^{s-1}}$. Thus the net present value of a perpetuity with growth contains three terms conceptually:

(A) E(1+g), the starting year's forecast earnings divided by r-g.

(B) $\frac{1}{r-g}$, which is the Gordon Model Multiple, which when multiplied by (A) gives us the net present value of the perpetuity as of the beginning of year s.

(C) $\frac{1}{(1+r)^{s-1}}$. Multiplying (A) $\times$ (B) by this term gives us the present value as of now, *i.e.*, Year 0. Note that when the first cash flow occurs at the end of Year 1, then s=1 and this term becomes 1, which leaves (A) $\times$ (B) unchanged.

For example, let's suppose we are valuing a business as of now that will begin operations three years from now. It will have a starting earnings of $1 million, grow at a perpetual rate of 5%, and the relevant discount rate, r, is 25%. The NPV is:

[7] $$NPV = \frac{\frac{1{,}000{,}000}{.25-.05}}{1.25^{3-1}}$$

The first denominator = 0.2. Dividing by .2 is the same as multiplying by 5, so term (1) = $5,000,000. In other words, as of the beginning of Year 3, the business will be worth $5 million. However, we must discount that value two years to come to the net present value as of now, the beginning of Year 1, or Year 0. The NPV is:

[8] $$NPV = \frac{5{,}000{,}000}{1.25^2}$$, or

[9] $$NPV = \frac{5{,}000{,}000}{1.5625} = \$3{,}200{,}000.$$

Midyear Cash Flows

Midyear cash flows are identical to end of year, but the midyear cash flows occur 6 months (one-half year) earlier. Therefore [6] also works for midyear cash flows, the only difference being that s will be smaller by 0.5 years.

Therefore, the Midyear Gordon Model formula is:

Gordon Model: Midyear Cash Flows

[10] $$NPV = \frac{\dfrac{E(1+g)}{(r-g)}}{(1+r)^{s-0.5}}$$

In valuing a stream of midyear cash flow beginning immediately, s = 0.5, which when substituted into [10] gives us a second denominator of $(1+r)^{-0.5}$. We can achieve the same effect by moving that term to the numerator and removing the minus sign, which gives us:

Gordon Model: Midyear Cash Flows Beginning Year 0

[11] $$NPV = E(1+g)\frac{\sqrt{1+r}}{(r-g)}$$

The first term is the forecast net income for the upcoming year, and the second term is the Gordon Model Multiple for a midyear cash flow.

Using the above example with the midyear Gordon formula instead of the end of year formula, the numerator is multiplied by $1.25^{0.5}$, which changes the NPV to $3,577,709.

The Gordon Model Is Approximately The Price/Earnings Ratio

Formula [11] gives us the expression for the fair market value (FMV) of a firm (before discounts). All we need do is substitute the term FMV for NPV. Also, the FMV of a firm is its price per share (P) multiplied by the number of shares (Sh). On the right hand side of equation [12], we use the right hand side of [11]. Therefore, we can restate [11] as:

[12] $$P \times Sh = E(1+g)\frac{\sqrt{1+r}}{(r-g)}$$

Next, we let earnings (in dollars) equal earnings per share (EPS) $\times$ the number of shares (Sh). Substituting that into [12] leads to [13]:

[13] $$P \times Sh = EPS \times Sh \times (1+g)\frac{\sqrt{1+r}}{(r-g)}$$

The shares cancel out, and dividing both sides of the equation by EPS, we have:

[13] $$\frac{P}{EPS} = (1+g)\frac{\sqrt{1+r}}{r-g}$$

The left hand term is the price earnings multiple, and the right hand term is the one-year forecast growth rate times the Gordon Model Multiple for a midyear cash flow beginning immediately. Renaming the price earnings multiple to the more familiar term PE, we finish our proof that the Gordon Model Multiple is a close approximation of the price earnings multiple.

[14] $$PE = (1+g)\frac{\sqrt{1+r}}{r-g}$$

The one-year forecast growth rate times the Gordon Model is an approximation of the PE multiple rather than an exact calculation, because real firms are always expected to grow at uneven rates in the early years, and the further out in time we forecast, the less confidence we have in our forecasts. We eventually forecast a constant growth from that point on. Therefore, the observed PE Multiple in the stock markets contains a growth rate that approximates the present value of the average growth throughout time, but it is inexact. Another reason that this is an approximation is that, most properly, we should be valuing a business based on its forecast cash flows, not earnings. We have used earnings in this article for simplicity.